Scaling behaviour of leptonic decay constants for heavy quarkonia and heavy mesons

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Abstract

In the framework of QCD sum rules one uses a scheme, allowing one to apply the conditions of both nonrelativistic heavy quark motion inside mesons and independence of nonsplitting nS-state density on the heavy quark flavours. In the leading order an analitic expression is derived for leptonic constants of both heavy quarkonia and heavy mesons with a single heavy quark. The expression allows one explicitly to determine scaling properties of the constants.

Introduction

By definition, heavy quarks have the m_Q mass values that are much greater than the confinement energy Λ. Therefore, in some cases, consideration of hadrons with the heavy quarks allows one to use expansions of some quantities over the small parameter of the Λ/m_Q ratio. If the heavy quark virtualities inside a hadron are not large, then one allows the kinematical expansion

$$
\begin{aligned}
p_Q^\mu &= m_Q \cdot v^\mu + k^\mu \,, \\
v \cdot k &\sim 0 \,, \\
|k^2| &\ll m_Q^2 \,,
\end{aligned}
\tag{1}
$$

where v is the quark 4-velocity, p_Q is the quark momentum, k characterizes the heavy quark virtuality. In the kinematics of eq.(1) in QCD, the heavy quark action, expanded over the small parameter, leads to Effective Heavy Quark Theory (EHQT) [1], so, in the leading approximation the theory possesses the symmetry with respect to the substitution of a heavy quark,

1

moving with the velocity $\vec{v}$, by any other heavy quark, moving with the same velocity $\vec{v}$ and having an arbitrary orientation of its spin.

In the system, where $v = (1, \vec{0})$, the heavy quark hamiltonian in an external field has the form

$$H = m_Q + V(r) + \frac{\vec{k}^2}{2m_Q} + g\frac{\vec{\sigma} \cdot \vec{B}}{2m_Q} + O(1/m_Q^2) \,, \tag{2}$$

where V is the potential, $\vec{B}$ is the chromomagnetic field.

In heavy mesons $(Q\bar{q})$ with a single heavy quark, one has

$$\frac{< \vec{k}^2 >}{m_Q} \quad \sim \quad O(1/m_Q) \,, \tag{3}$$

$$g\frac{< \vec{\sigma} \cdot \vec{B} >}{m_Q} \quad \sim \quad O(1/m_Q) \,,$$

and the distance r is determined by the light quark motion around the static source of the gluon field. Hence, in the leading approximation of EHQT, the heavy meson wave function is universal and independent of the flavour of the heavy quark inside the meson. This feature leads to both the scaling law for the leptonic constants of the heavy mesons

$$f^2 \cdot M = const. \,, \tag{4}$$

and the universality of form factors for semileptonic transitions between the hadrons, containing a single heavy quark (for example, $B \to D^{(*)} l\nu$) [1].

In the case of heavy quarkonium $(Q\bar{Q}')$, the chromomagnrtic field arises only at nonzero velocity of the source, so that

$$\vec{B} \sim O(\vec{v}) \sim O(1/m_Q) \,, \tag{5}$$

and, hence, spin-dependent splittings of the quarkonium levels arise only at the second order over $1/m_Q$, so that in what follows, we neglect the spin-dependent splittings in the heavy quarkonium.

As for the kinetic energy of the heavy quark motion inside the quarkonium $(Q\bar{Q}')$

$$T = \frac{\vec{k}^2}{2m_Q} + \frac{\vec{k}'^2}{2m_Q} \,, \tag{6}$$

it determines essentially both the quark binding energy E and the wave function $\Psi_E(\vec{r})$, as one can know this fact from an experience of working with nonrelativistic potential models of the quarkonia. Moreover, the distance between the heavy quarks inside the quarkonium depends on the quark masses. Thus, generally speaking, the leading approximation of EHQT may not be applied to the heavy quarkonium $(Q\bar{Q}')$, whose wave function depends on the quarkonium content.

However, as it has been shown in ref.[2], in the region of average distances between the heavy quarks inside the $(c\bar{c})$ charmonium and the $(b\bar{b})$ bottomonium

$$0.1 \; fm < r < 1 \; fm \; , \tag{7}$$

and with accuracy up to an additive shift, the QCD-motivated flavour-independent heavy quark potentials, behaving as the Coulomb interaction at small distances and having linearly rising confining part at large distances (Cornell model [3], Richardson potential [4], Buchmüller-Tye model [5]), allow the parameterizations in the forms of logarithmic [6] and power [7] laws, possessing simple scaling properties

$$V_L(r) \;\; = \;\; c_L + d_L \ln \Lambda r \; , \tag{8}$$
$$V_M(r) \;\; = \;\; -c_M + d_M(\Lambda r)^k \; . \tag{9}$$

By the virial theorem for average values of the kinetic energies in potentials (8), (9), one can get

$$< T_L > \;\; = \;\; d_L/2 = const. \; , \tag{10}$$
$$< T_M > \;\; = \;\; \frac{k}{k+2}(c_M + E) \; , \tag{11}$$

respectively, so that at $|E| \ll c_M$, $k \ll 1$ and with the accuracy by the small binding energy of the quarks inside the quarkonium, one concludes

$$< T_M > \simeq const. \tag{12}$$

In accordance with the Feynman-Hellmann theorem

$$\frac{dE}{d\mu} = -\frac{< T >}{\mu} \; , \tag{13}$$

3

Table 1: The Mass Difference (in MeV) for the Lightest Vector States with The Prescribed Valence Quark Contents.

state	Υ	ψ	ϕ
ΔM	563	588	660

where μ is the reduced mass of the heavy quarks $(Q\bar{Q}')$, and by condition (10), for the difference of the energies of two levels, one gets

$$E(\bar{n}, \mu) - E(n, \mu) = E(\bar{n}, \mu') - E(n, \mu') , \qquad (14)$$

i.e. the level density of the $(Q\bar{Q}')$ system does not depend on the heavy quark flavours

$$\frac{dn}{dM_n} = const. , \qquad (15)$$

that is rather accurately confirmed empirically [8] (see table 1).

In the framework of the QCD sum rules [9] the use of

1) the small parameter , $\Lambda/m_Q \ll 1$,

2) the nonrelativistic motion of the heavy quarks, $v \to 0$,

3) the universality of the quarkonium state density (15),

allows one, in the leading order,

1) to neglect power Λ/m_Q corrections from quark-gluon condensates,

2) to take into the account Coulomb-like interactions over α_S/v,

3) to derive the scaling relation for the leptonic decay constants of heavy S-wave quarkonium (see ref.[10])

$$\frac{f^2}{M} = const. , \qquad (16)$$

in the regime, when $|m_Q - m_{Q'}|$ is restricted at $m_{Q,Q'} \gg \Lambda$. Expression (16) is in a good agreement with the experimental values of f_Υ, f_ψ and f_ϕ.

4

In the present paper we generalize the analisys, made in ref.[10], for the regime of $m_Q = x m_{Q'} \gg \Lambda$ and derive the following expression, determining the scaling properties of the leptonic constants for the S-wave quarkonia

$$\frac{f^2}{M} \cdot \left(\frac{M}{4\mu}\right)^2 = const. \, ,$$

(17)

where μ is the reduced mass of the quarks.

Expression (17)

1) is reduced to eq.(16) at $x = 1$, i.e. at $4\mu/M \simeq 1$,

2) agrees with the scaling law for the leptonic constants of heavy $(Q\bar{q})$ mesons in the regime $M \to \infty$, $\mu = const.$, and

3) allows one to predict the f_{B_C} value for the heavy quarkonium B_c with the open charm and beauty (the B_c search is processed at LEP and FNAL).

In Section 1 the QCD sum rule scheme is considered. It allows one explicitly to use condition (15) and the dn/dM_n quantity as the phenomenological parameter, and to avoid unphysical dependence of results on the external parameters such as the number of the spectral density moment or the Borel transformation parameter. Expression (17) is derived.

In Section 2 one analyses the scaling relation (17), and in Conclusion the obtained results are discussed.

1 Quarkonium sum rules

Let us consider the two-point correlator functions of the quark currents

$$\Pi_{\mu\nu}(q^2) = i \int d^4 x e^{iqx} < 0|T J_\mu(x) J_\nu^\dagger(0)|0 > \, ,$$

(18)

$$\Pi_P(q^2) = i \int d^4 x e^{iqx} < 0|T J_5(x) J_5^\dagger(0)|0 > \, ,$$

(19)

where

$$J_\mu(x) = \bar{Q}_1(x)\gamma_\mu Q_2(x) \, ,$$

(20)

$$J_5(x) = \bar{Q}_1(x)\gamma_5 Q_2(x) \, ,$$

(21)

(22)

Q_i is spinor field of the heavy quark with $i = c, b$.

Further, write down

$$\Pi_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\Pi_V(q^2) + \frac{q_\mu q_\nu}{q^2}\Pi_S(q^2) , \qquad (23)$$

where Π_V and Π_S are the vector and scalar correlator functions, respectively. In the following we will consider the vector and pseudoscalar correlators: $\Pi_V(q^2)$ and $\Pi_P(q^2)$.

Define the leptonic constants f_V and f_P

$$< 0|J_\mu(x)|V(\lambda) > = i\epsilon_\mu^{(\lambda)} f_V M_V e^{ikx} , \qquad (24)$$

$$< 0|J_{5\mu}(x)|P > = ik_\mu f_P e^{ikx} , \qquad (25)$$

where

$$J_{5\mu}(x) = \bar{Q}_1(x)\gamma_5\gamma_\mu Q_2(x) , \qquad (26)$$

so that

$$< 0|J_5(x)|P >= i \frac{f_P M_P^2}{m_1 + m_2} e^{ikx} , \qquad (27)$$

where $|V >$ and $|P >$ are the state vectors of the 1^- and 0^- quarkonia, and λ is the vector quarkonium polarization, k is 4-momentum of the meson, $k_{P,V}^2 = M_{P,V}^2$.

Considering the charmonium (ψ, ψ' ...) and bottomonium (Υ, Υ', Υ'' ...), one can easily show that the relation between the width of the leptonic decay $V \to e^+e^-$ and f_V has the form

$$\Gamma(V \to e^+e^-) = \frac{4\pi}{9} e_i^2\alpha_{em}^2 \frac{f_V^2}{M_v} , \qquad (28)$$

where e_i is the electric charge of the quark i.

In the region of the narrow nonoverlapping resonances, it follows from Eqs.(18) - (27) that

$$\frac{1}{\pi}\Im m\Pi_V^{(res)}(q^2) = \sum_n f_{Vn}^2 M_{Vn}^2\delta(q^2 - M_{Vn}^2) , \qquad (29)$$

$$\frac{1}{\pi}\Im m\Pi_P^{(res)}(q^2) = \sum_n f_{Pn}^2 M_{Pn}^4 \frac{1}{(m_1 + m_2)^2}\delta(q^2 - M_{Pn}^2) , \qquad (30)$$

6

Thus, for the observed spectral function one has

$$\frac{1}{\pi}\Im m\Pi_{V,P}^{(had)}(q^2) = \frac{1}{\pi}\Im m\Pi_{V,P}^{(res)}(q^2) + \rho_{V,P}(q^2\mu_{V,P}^2) , \qquad (31)$$

where $\rho(q^2, \mu^2)$ is the continuum contribution, which is not equal to zero at $q^2 > \mu^2$.

Moreover, the operator product expansion gives

$$\Pi^{(QCD)}(q^2) = \Pi^{(pert)}(q^2) + C_G(q^2) < \frac{\alpha_S}{\pi}G^2 > +C_i(q^2) < m_i\bar{Q}_iQ_i > + \dots , \qquad (32)$$

where the perturbative contribution $\Pi^{(pert)}(q^2)$ is labled, and the nonperturbative one is expressed in the form of the quark-gluon condensate sum with the Wilson's coefficients, which may be calculated in the QCD perturbative theory.

In Eq.(32) we were restricted by the contribution of the vacuum expectation values for the operators with dimension $d = 4$. For $C_G^{(P)}(q^2)$ one has, for instance, [9]

$$C_G^{(P)} = \frac{1}{192m_1m_2}\frac{q^2}{\bar{q}^2}\left(\frac{3(3v^2+1)(1-v^2)^2}{2v^5}\ln\frac{1+v}{1-v} - \frac{9v^4+4v^2+3}{v^4}\right) , \quad (33)$$

where

$$\bar{q}^2 = q^2 - (m_1 - m_2)^2 , \quad v^2 = 1 - \frac{4m_1m_2}{\bar{q}^2} . \qquad (34)$$

The analogous formulae for other Wilson's coefficients can be found in Ref.[9]. In the following it will be clear that the explicit form of the coefficients has no significant meaning for the present consideration.

In the leading order of the QCD perturbation theory it was found for the imaginary part of the correlator that [9]

$$\Im m\Pi_V^{(pert)}(q^2) = \frac{\tilde{s}}{8\pi s^2}(3\bar{s}s - \bar{s}^2 + 6m_1m_2s - 2m_2^2s)\theta(s - (m_1+m_2)^2) \quad (35)$$

$$\Im m\Pi_P^{(pert)}(q^2) = \frac{3\tilde{s}}{8\pi s^2}(s - (m_1-m_2)^2)\theta(s - (m_1+m_2)^2) , \qquad (36)$$

where $\bar{s} = s - m_1^2 + m_2^2$, $\tilde{s}^2 = \bar{s}^2 - 4m_2^2s$.

The one-loop contribution into $\Im m\Pi(q^2)$ can be included into the consideration (see, for example, Ref.[9]). However, we note that the more essential

7

correction is that of summing a set over the powers of (α_s/v), where v is defined in Eq.(34) and is a relative quark velocity, and α_S is the QCD interaction constant. In Ref.[9] it has been shown that the account of the coulomb-like gluonic interaction between the quarks leads to the factor

$$F(v) = \frac{4\pi}{3} \frac{\alpha_S}{v} \frac{1}{1 - \exp(-\frac{4\pi\alpha_S}{3v})} \ , \qquad (37)$$

so that the expansion of the $F(v)$ over $\alpha_S/v \ll 1$ restores, precisely, the one-loop $O(\frac{\alpha_S}{v})$ correction

$$F(v) \approx 1 - \frac{2\pi}{3} \frac{\alpha_s}{v} \ \ldots \qquad (38)$$

In accordance with the dispersion relation one has the QCD sum rules, which state that, in average, it is true that, at least, at $q^2 < 0$

$$\frac{1}{\pi} \int \frac{\Im m \Pi^{(had)}(s)}{s - q^2} ds = \Pi^{(QCD)}(q^2) \ , \qquad (39)$$

where the necessary subtractions are omitted. $\Im m\Pi^{(had)}(q^2)$ and $\Pi^{(QCD)}(q^2)$ are defined by Eqs.(29) - (31) and Eqs.(32) - (38), respectively. Eq.(39) is the base to develop the sum rule approach in the forms of the correlator function moments and of the Borel transform analysis (see Ref.[9]). The truncation of the set in the right hand side of Eq.(39) leads to the mentioned unphysical dependence of the $f_{P,V}$ values on the external parameter of the sum rule scheme.

Further, let us use the conditions, simplifying the consideration due to the heavy quarkonium.

1.1 Nonperturbative Contribution

We assume that, in the limit of the very heavy quark mass, the power corrections of the nonperturbative contribution are small. From Eq.(33) one can see that, for example,

$$C_G^{(P)}(q^2) \approx O(\frac{1}{m_1 m_2}) \ , \ \ \Lambda/m_{1,2} \ll 1 \ , \qquad (40)$$

8

where v is fixed, $q^2 \sim (m_1 + m_2)^2$, when $\Im m \Pi^{(pert)}(q^2) \sim (m_1 + m_2)^2$. It is evident that, due to the purely dimensional consideration, one can believe that the Wilson's coefficients tend to zero as $1/m_{1,2}^2$.

Thus, the limit of the very large heavy quark mass implies that one can neglect the quark-gluon condensate contribution.

1.2 Nonrelativistic Quark Motion

The nonrelativistic quark motion implies that, in the resonant region, one has, in accordance with Eq.(34), that

$$v \to 0 . \tag{41}$$

So, one can easily find that in the leading order

$$\Im m \Pi_P^{(pert)}(s) \approx \Im m \Pi_V^{(pert)}(s) \to \frac{3v}{8\pi^2} s \left(\frac{4\mu}{M} \right)^2 , \tag{42}$$

so that with account of the coulomb factor

$$F(v) \simeq \frac{4\pi}{3} \frac{\alpha_S}{v} , \tag{43}$$

one obtaines

$$\Im m \Pi_{P,V}^{(pert)}(s) \simeq \frac{\alpha_S}{2} s \left(\frac{4\mu}{M} \right)^2 . \tag{44}$$

1.3 "Smooth Average Value" Scheme of the Sum Rules

As for the hadronic part of the correlator, one can write down for the narrow vector resonance contribution

$$\Pi_V^{(res)}(q^2) = \int \frac{ds}{s - q^2} \sum_n f_{Vn}^2 M_{Vn}^2 \delta(s - M_{Vn}^2) , \tag{45}$$

$$\Pi_P^{(res)}(q^2) = \int \frac{ds}{s - q^2} \sum_n f_{Pn}^2 \frac{M_{Pn}^4}{(m_1 + m_2)^2} \delta(s - M_{Pn}^2) , \tag{46}$$

The integrals in Eqs.(45)-(46) are simply calculated, and this procedure is generally used.

In the presented scheme, let us introduce the function of the state number $n(s)$, so that

$$n(m_k^2) = k \ . \tag{47}$$

This definition seems to be reasonable in the resonant region. Then one has, for example, that

$$\frac{1}{\pi} \ \Im m \Pi_V^{(res)}(s) = s f_{Vn(s)}^2 \frac{d}{ds} \sum_k \theta(s - M_{Vk}^2) \ . \tag{48}$$

Further, it is evident that

$$\frac{d}{ds} \sum_k \theta(s - M_k^2) = \frac{dn(s)}{ds} \frac{d}{dn} \sum_k \theta(n - k) \ , \tag{49}$$

and Eq.(45) may be rewritten as

$$\Pi_V^{(res)}(q^2) = \int \frac{ds}{s - q^2} \ s f_{Vn(s)}^2 \frac{dn(s)}{ds} \frac{d}{dn} \sum_k \theta(n - k) \ . \tag{50}$$

The "smooth average value" scheme means that

$$\Pi_V^{(res)}(q^2) = < \frac{d}{dn} \sum_k \theta(n - k) > \int \frac{ds}{s - q^2} s f_{Vn(s)}^2 \frac{dn(s)}{ds} \ . \tag{51}$$

It is evident that, in average, the first derivative of the step-like function in the resonant region is equal to

$$< \frac{d}{dn} \sum_k \theta(n - k) > \simeq 1 \ . \tag{52}$$

Thus, in the scheme one has

$$< \Pi_V^{(res)}(q^2) > \ \approx \ \int \frac{ds}{s - q^2} s f_{Vn(s)}^2 \frac{dn(s)}{ds} \ , \tag{53}$$

$$< \Pi_P^{(res)}(q^2) > \ \approx \ \int \frac{ds}{s - q^2} \frac{s^2 f_{Pn(s)}^2}{(m_1 + m_2)^2} \frac{dn(s)}{ds} \ . \tag{54}$$

Eqs.(53)-(54) give the average correlators for the vector and pseudoscalar mesons, therefore, due to Eq.(39) we state that

$$\Im m < \Pi^{(hadr)}(q^2) > = \Im m \Pi^{(QCD)}(q^2) \ , \tag{55}$$

that gives with account of Eqs.(44), (53) and (54) at the physical points $s_n = M_n^2$

$$\frac{f_n^2}{M_n} = \frac{\alpha_S}{\pi} \frac{dM_n}{dn} \left(\frac{4\mu}{M}\right)^2 , \tag{56}$$

where in the limit of the heavy quarks we use, that for the lightest resonances one has

$$m_1 + m_2 \approx M , \tag{57}$$

so that

$$f_{Vn} \simeq f_{Pn} = f_n . \tag{58}$$

Thus, one can conclude that the QCD sum rules give for the heavy quarkonia the identity of the f_P and f_V values for the lightest pseudoscalar and vector states.

Eq.(56) differs from the ordinary sum rule scheme because it does not contain the parameters, which are external to QCD. The quantity dM_n/dn is purely phenomenological. It defines the average mass difference between the nearest levels with the identical quantum numbers.

Note, in the Borel sum rules, the derivative procedure over σ gives the possibility to find both the constants f and the bound state mass versus the current quark mass choice.

It must be noted, that the approximation made implies that we neglected the continuum contribution in the resonant region, and this assumption is valid for the lightest states only.

The relations, connecting f with the dM_n/dn value, were derived in some other ways, so in the quasiclassical approximation [11], in the sum rule analysis with the use of the Euler-McLohren transformation [12] and by the double action of the Borel transformation [13].

2 Sum rule analysis

As it has been noted in Introduction, the phenomenological properties of the heavy quark potential lead to the independence of the quarkonium state density on the heavy quark flavours (see eq.(15)). Thus, in accordance with eq.(56) and in the leading approximation with no account of the logarithmic and power corrections, one can draw the conclusion, that for the leptonic

Table 2: The Experimental Values of the Leptonic Constants (in MeV) for the Quarkonia in Comparison with the Estimates of Present Model.

quantity	exp.	present
f_ϕ	232 ± 5	230 ± 25
f_ψ	409 ± 13	400 ± 40
f_Υ	714 ± 14	700 ± 70

constants of the S-wave quarkonia the scaling relation takes place

$$\frac{f^2}{M} \left(\frac{M}{4\mu}\right)^2 = const. \ , \tag{59}$$

independently of the heavy quark flavours, so that the constant in the right hand side of eq.(59) is determined by the expression

$$const. = \frac{\alpha_S}{\pi} \frac{dM_n}{dn} \ . \tag{60}$$

In ref.[10], for the numerical estimate of the constant we have used the data on the average mass difference of the S-wave bottomonium

$$< \frac{dM}{dn} > = \frac{1}{2}((M_{\Upsilon'} - M_\Upsilon) + (M_{\Upsilon''} - M_{\Upsilon'})) \ , \tag{61}$$

and the flavour-independent value of α_S, determining the Coulomb term of the Cornel potential,

$$\alpha_S \simeq 0.36 \ . \tag{62}$$

In the case of the quarkonia with the hidden flavour, one has $4\mu/M = 1$. So, the calculated values of the leptonic constants for the Υ-, ψ- and ϕ-mesons are presented in table 2, and they are in a good agreement with the experimental values of these quantities.

For the heavy $(\bar{b}c)$ quarkonium with the open charm and beauty, one can easily find the estimate

$$f_{BC} = 460 \pm 60 \ MeV \ , \tag{63}$$

where the uncertainty is caused by the ambiguity in the choice of the quark masses [9]

$$m_c = 1.4 \div 1.8 \ GeV \ , \tag{64}$$
$$m_b = 4.6 \div 5.2 \ GeV \ . \tag{65}$$

In the potential models [15, 16, 17, 18], the mass estimates for the S-wave levels of the B_c mesons agree with the expected behaviuor, when the mass difference is practically the same, as for the families of the charmonium and the bottomonium.

The f_{B_C} value (63) is in an agreement with the other estimates, obtained in the framework of both the potential models [15, 16, 17, 19, 20, 21, 22] and the QCD sum rules [14, 16, 18, 23], where, in the other schemes, a large spread in the f_{B_C} predictions takes place due to the ambiguity in the choice of the hadronic continuum threshold, the number of the spectral density momentum or the Borel parameter.

Further, in the limit case of B- and D-mesons, when the heavy quark mass is much greater than the light quark mass $m_Q \gg m_q$, one has

$$\mu \simeq m_q$$

and

$$f^2 \ M = \frac{16\alpha_S}{\pi} \ \frac{dM}{dn} \ \mu^2 \ . \tag{66}$$

Then it is evident that at one and the same μ one gets

$$f^2 \ M = const. \tag{67}$$

Scaling law (67) is very well known in EHQT [1] for mesons with a single heavy quark $(Q\bar{q})$, and it follows, for example, from the identity of the B- and D-meson wave functions in the limit, when infinitely heavy quark can be considered as a static source of gluon field.

In our derivation of eqs.(66) and (67) we have neglected power corrections over the inverse heavy quark mass. Moreover, we have used the presentation about the light constituent quark with the mass, equal to

$$m_q \simeq 330 \ MeV \ , \tag{68}$$

so that this quark has to be considered as nonrelativistic one $v \to 0$, and the following conditions take place

$$m_Q + m_q \approx M_{(Q\bar{q})}^{(*)} , \quad m_q \ll m_Q , \tag{69}$$

and

$$f_V \simeq f_P = f . \tag{70}$$

In agreement with eqs.(66) and (68), one finds the estimates[1]

$$f_{B^{(*)}} = 120 \pm 20 \ MeV , \tag{71}$$

$$f_{D^{(*)}} = 220 \pm 30 \ MeV , \tag{72}$$

that is in an agreement with the estimates in the other schemes of the QCD sum rules [24].

Thus, in the limits of $4\mu/M = 1$ and $\mu/M \ll 1$, scaling law (59) is consistent.

In ref.[14] the sum rule scheme with the double Borel transform has been used, so, it allows one to study effects, related to power corrections from the gluon condensate, corrections due to nonzero quark velocity and nonzero binding energy of the quarks in the quarkonium.

The numerical effect from the mentioned corrections considers to be not large (the power corrections are of the order of 10%), and the uncertainty, connected to the choice of the quark masses, dominates in the error of the f_{B_C} value determination.

Conclusion

In the present paper, in the framework of the QCD sum rules and in the leading approximation, we have considered the scaling properties of the leptonic constants for the S-wave quarkonia with heavy quarks, and in the specific scheme, allowing one to use the spectroscopic data on the quarkonium level density, we have gotten the relation

$$\frac{f^2}{M} \left(\frac{M}{4\mu} \right)^2 = const. ,$$

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[1]In ref.[10] the dependence of the S-wave state density dn/dM_n on the reduced mass of the system with the Martin potential has been found by the Bohr-Sommerfeld quantization, so that at the step from $(\bar{b}b)$ to $(\bar{b}q)$, the density changes less than about 15%.

that is in the good agreement with the experimental data on the leptonic constants for the Υ-, ψ- and ϕ-mesons. It allows one to predict the f_{B_C} value for the B_c meson, whose search is processed at LEP and FNAL.

The estimates for the leptonic constants of the B and D mesons due to the scaling relation is in the agreement with the values, obtained in the framework of the other schemes of the QCD sum rules.

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